Comparative Study about the Damping Properties of the Sandwich Beams with Core by Polystyrene or Polypropylene Honeycomb

DUMITRU BOLCU[1], MARIUS MARINEL STANESCU[2]*, ION CIUCA[3], COSMIN MIHAI MIRITOIU[1], RAUL CORMOS[4]

[1] University of Craiova, Department of Mechanics, 165 Calea Bucureşti, 200620, Craiova, Romania

[2] University of Craiova, Department of Applied Mathematics, 13 A.I. Cuza, 200396, Craiova, Romania

[3] Politehnica University of Bucharest, Department of Materials Science and Engineering, 313 Splaiul Independentei, 060032, Bucharest, Romania

[4] Politehnica University of Bucharest, Department of Strenght of Materials, 313 Splaiul Independentei, 060032, Bucharest, Romania

Starting from the dynamic response of a sandwich beam with damping (which is in free vibration), is established a method used to determine the damping factor. We experimentally calculated the stiffness and damping factor per unit length and unit mass for beams with middle layer made of polystyrene and polypropylene honeycomb respectively; the external layers were made of epoxy resin reinforced with fiber-glass fabric and carbon-fiber fabric respectively.

Keywords: sandwich beam, damping factor, polystyrene, polypropylene honeycomb, free vibrations.

Vibration analysis for Timoshenko type beams continues to be of interest to engineers since the beginning of last century. There are several studies [1-11] which dwell on the effects of shearing force and the rotational inertia upon the beam frequencies and also a few notable studies relating to the forced vibration analysis of Timoshenko type beams with general boundary conditions.

In [12] was analyzed the axial load and damping for a Timoshenko beam for random transversal loads. It was considered just a special damping case in rotation and tranversal movements. This fact allowed using the orthogonality conditions of the undamped modes for decoupling of modal equations.

A closed-form solution, for incomplete differential equations of the simply supported beam, with external damping, was obtained in [13]. The authors have neglected eight terms from complete differential equation of the Timoshenko beam. Three of these terms were obtained from disturbing forces (external).

The paper [14] presents a general modal approach for solving the problem of linear vibrations of the Timosenko beam, with constant section, having the viscous damping of rotation and transversal external and different viscoelastic damping in shearing and bending deformations. With this approach the beams with different boundary conditions can be analyzed as simply supported beams. Using this approach, the numerical results of the response are obtained and reported for transversal loads, correlated and uncorrelated, temporal and spatial, applied to beam.

In [15] was analyzed the dynamic response of a general class of continuous linear vibrating systems which have damping properties close to those resulting from normal modes classically (decoupled). First, the conditions are given for the existence of classical modes of vibration in a continuous linear system, special attention being given to boundary conditions.The extensions of periodic perturbations (in terms of forms of the undamped mode) are used for analyzing the eigen problem as well as response of vibrations, for nearly all classical damped systems. The analysis is based on a proper splitting of the damping operators in both the field equations and the boundary conditions.

The purpose of research done in [16], was to investigate the curvature, and the face / core debonding influence on the vibration behaviour of curved composite sandwich beams built from carbon / epoxy laminate skins over a foam polyurethane core. The sandwich-type beams, flat and curved, with debonding, were prepared by keeping of the arc length of the sandwich beams, equal to the length of flat sandwich beams. Natural frequencies and damping loss factor, for sandwich beams, were determined using impulse frequency response technique under free-free boundary conditions.

The paper [17] presents a numerical investigation of changes to natural frequencies due to the separations along the line of solder in a composite sandwich beam (as a fascicle of stratified fiber). The numerical analysis is made using the link elements (rigid links and master slave links). The analysis is done using finite element method.

Recent applications have shown that honeycomb panels from polymer, reinforced with fiber, can be used for new construction or for restoration of existing structures. In [18] are studied the vibrations of sandwich structures with honeycomb which have the core geometry of sinusoidal type. It was developed a higher order vibration model for studying the vibrations, made by energy methods.

In [19] were investigated the free vibrations of the curved sandwich beams, with flexible core, in different conditions of temperature. The external surfaces and the core of the beam were considered as being made of materials with mechanical properties dependent on temperature. It was shown that the frequency of free vibrations of the beams decreases when the temperature increases.

Theoretical aspects

The movement equations for the transversal vibrations of the viscoelastic beams, with constant section, and

* *email: mamas1967@gmail.com*

external damping, are presented in [14]. It is shown that if the ratio between the beam length and the beam width is larger than ten, the difference between the Timoshenko and Euler Bernoulli theories, for bending moment, the shearing force and the deformation of medium fibre, are less than 5 %. It is shown that for the first eigenmodes of vibrations, the damping influence to the rotational movement of the beam section, can be neglected. The equations and similar conclusions for beam made of composite materials are presented in [20, 21].

In the case of small deformations, the equation of motion for transversal vibrations of the beam is:

$$\langle \rho A\rangle \ddot{w} + c^* \dot{w} + \langle EI\rangle \frac{\partial^4 w}{\partial x^4} = p(x,t), \qquad (1)$$

where

- w(x, t) is the transversal displacement, for the elastic center of the beam section;

$$\langle \rho A\rangle = \iint_{(S)} \rho(x,y)\,dS\,;$$

$$\langle EI\rangle = \iint_{(S)} E y^2\,dS\,;$$

- ρ(x, y) is the density of beam material;
- E(x, y) - Young's modulus of the material;
- p(x, t) - the transversal load, applied to beam;
- c*- the damping coefficient per unit length of the beam.

The damping determination can be performed by studying the free vibrations (p(x,t)=0), produced by an initial deformation of the beam, i.e the initial conditions:

$$w(x,0) = f(x); \quad \dot{w}(x,0) = 0. \qquad (4)$$

The beam vibration, in this case, has the expresion:

$$w(x,t) = \sum_{n=1} p_n \cdot e^{-\frac{c}{2}t}\left(\frac{c}{2\omega_n}\sin\omega_n t + \cos\omega_n t\right)\cdot v_n(x), \qquad (5)$$

in which

- $c = \frac{c^*}{\langle \rho A\rangle}$ is the damping coefficient per unit mass of the beam;
- $\nu_n(x)$ are the eigenfunctions which depend on the conditions of the beam ends;

$$- \; p_n = \int_0^L f(x)\cdot v_n(x)\,dx\,; \qquad (6)$$

- ω_n are eigenpulsations of the beam:

$$\omega_n = \sqrt{\lambda_n^4 \frac{\langle EI\rangle}{\langle \rho A\rangle} - \frac{c^2}{4}}\,; \qquad (7)$$

- λ_n are determined from supported conditions of the beam.

If the beam is embedded in x = 0 and free in x = L, the eigenfunctions have the form:

$$v_n(x) = C_n\left(U(\lambda_n x) - \frac{S(\lambda_n L)}{T(\lambda_n L)}\cdot V_n(\lambda_n x)\right), \qquad (8)$$

where

- S(λx), V(λx), U(λx) and T(λx) are the Krîlov functions, and C_n is determined from the orthogonality conditions

$$\int_0^L v_n(x)\cdot v_k(x)\,dx = \begin{cases} 1, & if\ n = k; \\ 0, & if\ n \neq k. \end{cases} \qquad (9)$$

The experimental record of free-vibrations gives the posibility of the damping and the beam rigidity characteristics calculus. After we have experimentally determined the beam vibration in one point, we can calculate the damping coefficient as follows:

-there are determined the time-values where the displacement is zero (points where the graph intersects the time axis);

-it is determined the cancellation period of the movement, more precisely T is the double time interval between two successive cancellation;

-it is determined the frequency $\upsilon = \frac{1}{T}$ and the pulsation $\omega = \frac{2\pi}{T}$;

-it is determined the damping factor

$$\frac{c}{2} = \frac{1}{T}\ln\frac{A_i}{A_{i+1}} = \frac{\omega}{2\pi}\ln\frac{A_i}{A_{i+1}}, \qquad (10)$$

where A_i and A_{i+1} are the values for two successive points (by maximum), of the graph.

Results and discussions

We have made plates with core by polystyrene and polypropylene honeycomb, with 20 mm thickness. The exterior layers were made of epoxy resin reinforced with fiber-glass and carbon-fiber fabric. From these plates, we have taken samples with length 400 mm and width 50 mm and 40 mm For determining the rigidities, we subject the samples to bending (fig. 1).

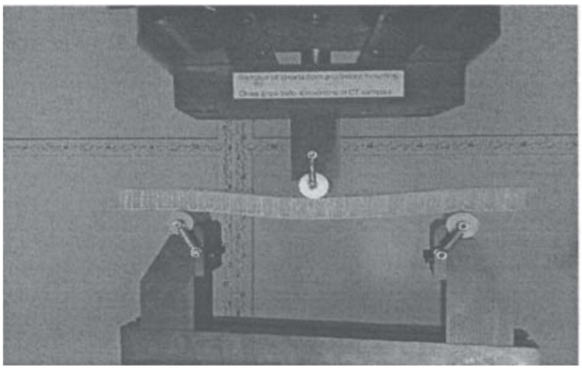


Fig. 1. Testing device

Samples and test device were configured according to ASTM D790-02, Standard Test Methods for Flexural Properties of Unreinforced and Reinforced Plastic and Electrical Insulating Materials. The test speed has been calculated according to ASTM D790-02.

Specifications of the test device are:

-the radius of support and identor rollers - 25 mm

-the distance between the support rollers - 2 x 120 mm

-the width of support and identor rollers - 50 mm.

In figures 2 – 3 it is presented the dependence between the force applied by the identor roller and the displacement at the middle of the sample.

The analysis of these diagrams highlights the existance of three areas:

-a first area where there is a proportionality between force and deformation (displacement) corresponding to an elastic area of loading;

-an area where the displacement is very high although the force remains relatively constant; this fact is explained by the upper layer deterioration of the sample in the contact area with the identor roller;

-an area where the displacement increases although the force decreases, area where the fibres breaking from the exterior layers is produced.

The specific mass ⟨ρA⟩ was determined by weighing the samples. The specific mass ⟨ρA⟩ is obtained by dividing the mass of the sample to its length.

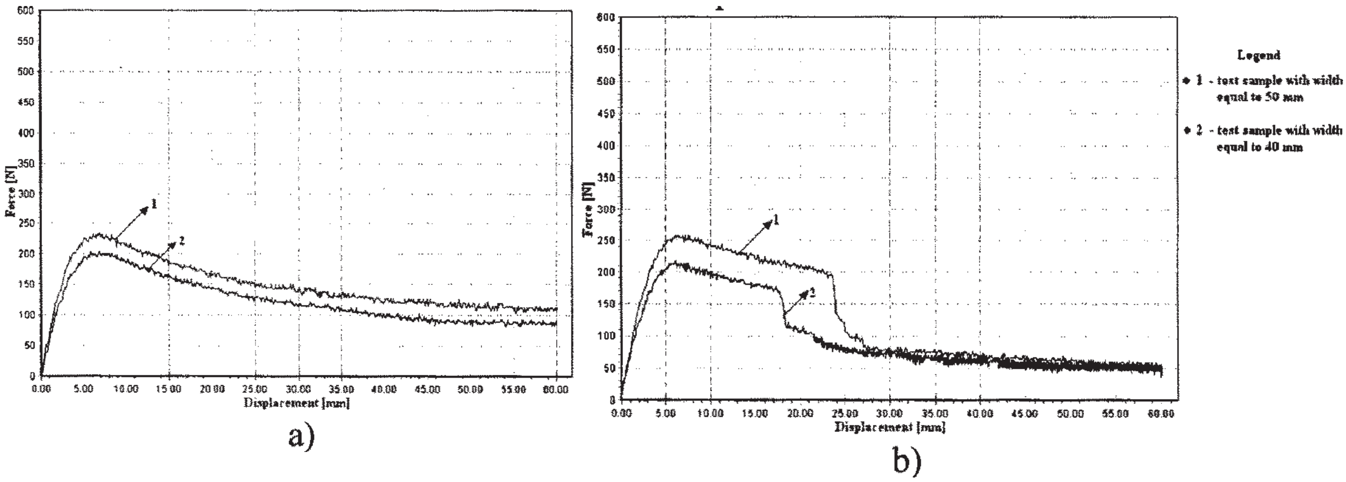


Fig. 2. The dependence between the force applied by the indentor roller and the displacement at the middle of the sample, for samples with polystyrene core, the reinforcement of the exterior layers being made with:
a) fiber-glass fabric; b) carbon-fiber fabric

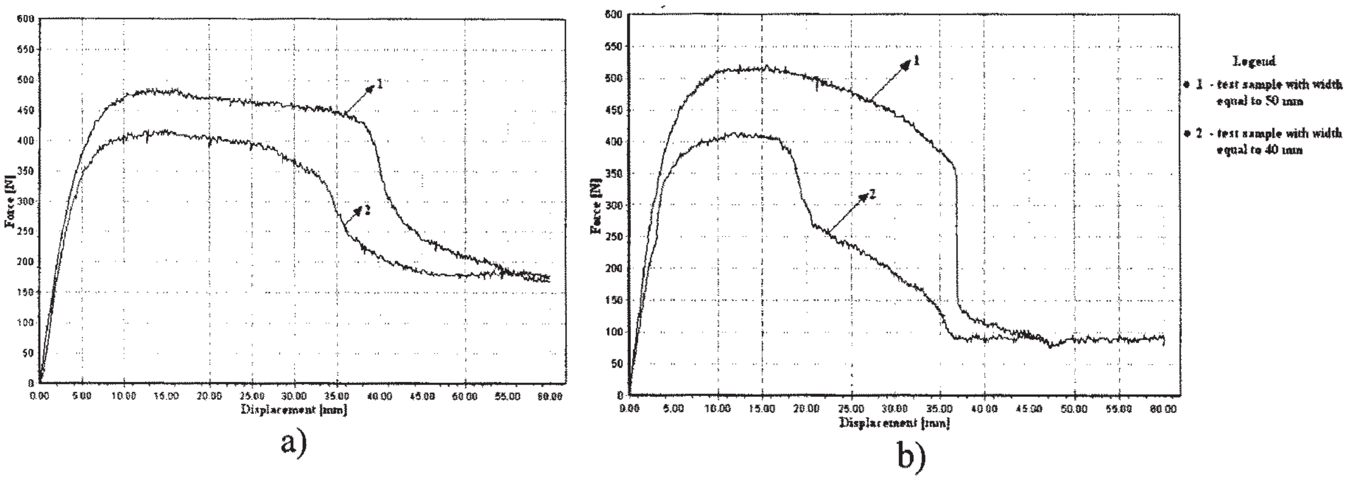


Fig. 3. The dependence between the force applied by the indentor roller and the displacement at the middle of the sample, for samples with core from polypropylene honeycomb, the reinforcement of the exterior layers being made with:
a) fiber-glass fabric; b) carbon-fiber fabric

The ratios between the specific masses of the samples with 40 mm width and the specific masses of the ones with 50 mm width are:

-for the samples with core made of polystyrene and exterior layers made of epoxy resin reinforced with fiber-glass fabric - 0.792;

- for the samples with core made of polypropylene honeycomb and exterior layers made of epoxy resin reinforced with fiber-glass fabric - 0.812;

- for the samples with core made of polystyrene and exterior layers made of epoxy resin reinforced with carbon-fiber fabric - 0.818;

- for the samples with core made of polypropylene honeycomb and exterior layers made of epoxy resin reinforced with carbon-fiber fabric - 0.815;

These ratios are near 0.8 which represent the ratio between the samples widths, the small deflections are explained by the irregularities in the materials distribution from the sample composition.

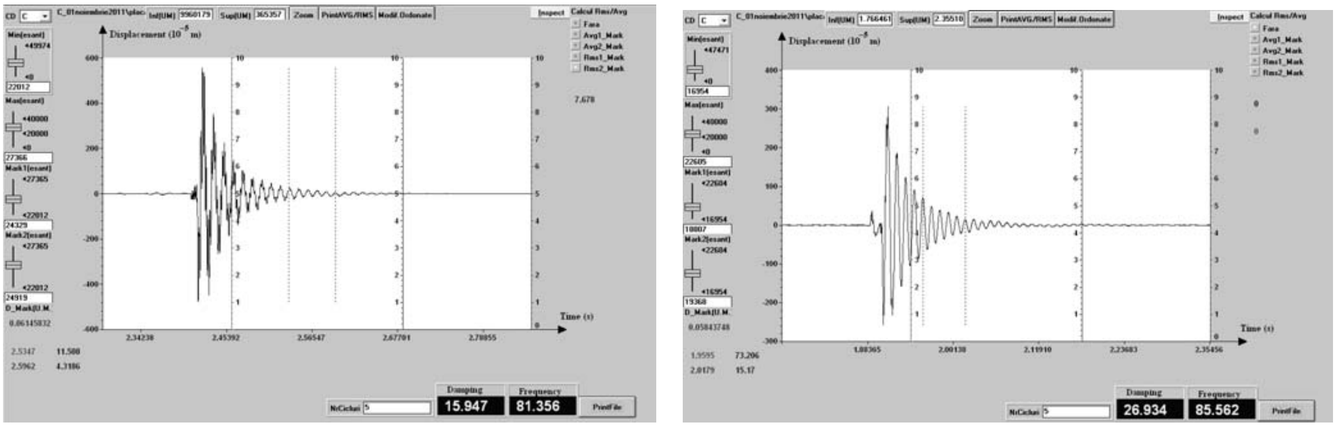
The samples reinforced with fiber-glass fabric have a higher specific mass than the ones reinforced with carbon-fiber fabric. This is explained by the fact that the fiber-glass fabric specific mass is higher than the one from the carbon-fiber fabric and by the fact that is was necessary to use more resin for the samples with the exterior layers fiber-glass fabric reinforced.

The rigidities of samples $\langle EI \rangle$ were calculated by determining the deformations produced in proportionality zone by the applied force of identor roller. So, in the case of a simply supported bar, middle-loaded by a concentrated force, for loadings in the elastic domain, the ratio between the loading force and the middle-bar deformation (displacement) is proportional with the rigidity $\langle EI \rangle$ and inverse proportional with the cubic distance between the supports. So, by determining the ratio between the force and the displacement at the middle of the bar, the bar rigidity $\langle EI \rangle$ can be calculated. In the figures 2 - 3 there are presented the dependencies between the force applied at the middle of the bar and the bar displacements in the point of the force appliance. From each of these diagrams it is observed that for deformations (displacements) lower than 3 mm there is proportionality between force and displacement. This area was used for bar rigidity calculus. The specific mass $\langle \rho A \rangle$ depends on the sample dimensions and the mass properties of compound materials. In addition, the rigidity depends on the three dimensional distribution of compound materials. This is why there is not proportionality between the specific masses and bar rigidities. For each sample, there were considered ten values for which the considered rigidities were calculated, taking their average value.

To determine the damping coefficient the free-vibration were measured for the 8 sets of samples. The beams were embedded (keeping in console the length 0.256 mm). These were deformed by a force applied to the free end and then were left to vibrate freely. Because the form of deformed medium fiber (this deformation was produced

Sample No.	Core material	Reinforcement Material	Width Sample (mm)	$\langle \rho A \rangle$ kg/m	$\langle EI \rangle$ N.m^2
1	Polystyrene	Fiber-glass fabric	50	0.216	21.438
2	Polystyrene	Fiber-glass fabric	40	0.171	18.454
3	Polypropylene honeycomb	Fiber-glass fabric	50	0.282	36.314
4	Polypropylene honeycomb	Fiber-glass fabric	40	0.229	30.988
5	Polystyrene	Carbon-fiber fabric	50	0.176	21.730
6	Polystyrene	Carbon-fiber fabric	40	0.144	18.383
7	Polypropylene honeycomb	Carbon-fiber fabric	50	0.238	41.111
8	Polypropylene honeycomb	Carbon-fiber fabric	40	0.194	36.973

Table 1
MASS AND RIGIDITY CHARACTERISTICS, EXPERIMENTALLY CALCULATED FOR THE SETS OF THE USED SAMPLES



a b

Fi. 4. Experimental recording of damped vibration for a bar with 40 mm width, with core made of polystyrene and exterior layers made of epoxy resin reinforced with: fiber-glass fabric a); carbon-fiber fabric b)

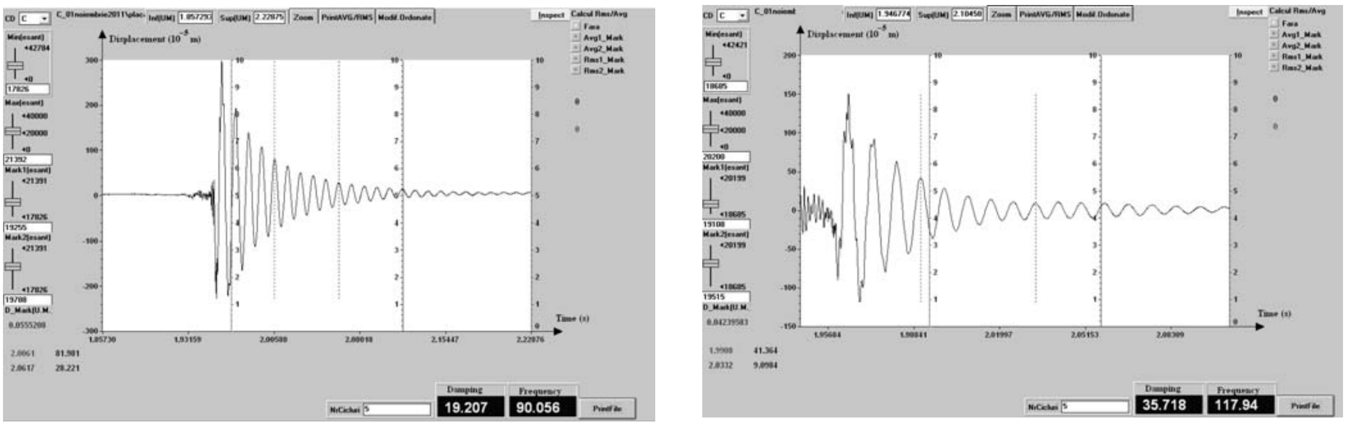


Fig. 5. Experimental recording of damped vibration for a bar with 40 mm width, with core made of polypropylene honeycomb and exterior layers made of epoxy resin reinforced with: fiber-glass fabric a); carbon-fiber fabric b)

by external force) is similar to its first eigen mode of vibration, the measured frequency is considered the first eigen frequency.

In figures 4 - 5 are presented the results of the experimental records of damped vibration for four test-samples (representative), with width 40 mm.

The damping coefficient per unit mass for each sample has been calculated by using the relation (10), for five consecutive maximum of the diagrams and taking the mean value.

The five maximals used for damping factor calculus are delimited in figures 4 – 5 by the two doted paralel lines, drawn inside each graphic.

These maximals have been chosen after the transitory area, avoiding the vibration eigenmodes influences.

The damping factors and the eigenfrequencies of the first vibration mode are presented in figures 4 – 5.

The experimental results for all the 8 sets are presented in table 2.

Table 2
EXPERIMENTAL RESULTS

Sample No.	υ_{exp} (s^{-1})	υ_{teor} (s^{-1})	$\frac{c}{2}$ $\left(\frac{Ns/m}{kg}\right)$	c^* $\left(\frac{Ns/m}{m}\right)$
1	76.43	78.89	19.169	8.358
2	81.36	82.73	15.947	5.454
3	90.39	90.36	20.987	11.836
4	90.06	91.13	19.207	8.797
5	85.26	88.40	29.096	10.242
6	85.26	89.93	26.934	7.584
7	108.11	104.52	39.974	19.028
8	117.94	112.79	35.718	13.858

The damping coefficient per unit length has been calculated by multiplying the damping coefficient per unit mass with linear specific mass of the sample material.

The theoretical frequency has been calculated for the first eigenpulsation by using the linear specific mass ($\langle\rho A\rangle$) and rigidity ($\langle EI\rangle$), determined experimentally and presented in table 1.

Conclusions

From the diagrams which give the dependence between the force applied in the middle of beam and the displacement measured in the same point, we can determine the equivalent stiffness of the beam section. The results obtained show (as expected) that the beams with width 40 mm have the stiffness smaller than the beams with width 50 mm (for the same type of core). It is not respected a strict proportionality (in the case in which the core is from polypropylene honeycomb) because the rigidity depends, among other, by how the cutting section divides the cell of honeycomb from which the median part of samples is built.

It is observed that the samples rigidities which are reinforced with carbon-fiber fabric are higher than the samples rigidities which are reinforced with fiber-glass fabric (regardless of core: polypropylene honeycomb or polystyrene).

This can be explained based on the following arguments:

-the elasticity modulus of the carbon fiber is much higher than the elasticity modulus of the fiber glass;

-the elasticity modulus of the exterior layers of the samples (to same thickness of the layers) is higher when those are made of epoxy resin reinforced with carbon fiber than to those made up of epoxy resin reinforced with fiber glass.

In conclusion, the stiffness of samples (with the same type of core) will be higher when reinforcement of the exterior layers is made of carbon fiber than when it is made of fiberglass.

The maximum values of the applied forces at the request of bending, keep the proportionality with the width of sample. As with rigidity, the maximum forces of load are higher to samples with the middle from polypropylene honeycomb than to samples with the middle from polystyrene.

Both for samples reinforced with carbon fiber as well as for those reinforced with fiberglass, the maximum value of the force applied to samples with core by honeycomb is almost twice the maximum value of the force applied to samples with core by polystyrene. A close value is also obtained for the ratios between the rigidities of samples with core from polypropylene honeycomb and rigidities of samples which have the core from polystyrene.

In terms of the rigidities, the samples with core from polypropylene honeycomb have a higher rigidity than the samples with core from polystyrene and the samples reinforced with carbon fiber are superior than the samples reinforced with fiberglass.

Similar conclusions are obtained for the damping coefficient. Both for damping factor per unit length of the beam, and for damping factor per unit mass of the beam, the following conclusions are true:

-the beams with 50 mm width have the damping factor higher than the beams with 40 mm width (for the same type of core and reinforcement);

-the beams with polypropylene honeycomb core have the damping factor higher than the beams with polystirene core (for the same type of reinforcement);

-the beams which have the exterior layers reinforced with carbon fiber have the damping factor higher than the beams which have the exterior layers reinforced with fiberglass (for the same type of core).

The values analysis of damping factors indicate that these factors must be experimentally determined for each type of material and sample, being difficult to deduce a quantitative correspondence with the parameters which influence the damping directly or indirectly. The values of damping factors may depend on several features such as: sample dimensions, specific mass or the quantity of material from sample, elastic and damping properties of component materials.

The sample width can influence the damping coefficient, by the fact that it determines the surface in which the air friction acts on the sample. The sample mass or specific linear mass influence the damping factor by the fact that the samples with higher mass and width, the deformation energy which is stored in sample through the initial deformation, is dissipated in a larger quantity of material. An influence may occur due to the sample rigidity, explained by the fact that a force initially applied on the sample produces a smaller deformation if the rigidity is higher.

A good damping of vibrations is achieved in the case in which the composite materials of the external layers, have higher damping capacity and elastic properties (such as the carbon fiber against fiber glass). But the influence of these layers is dependent on the interaction with the middle layer and for this reason, it is difficult to be analytically analyzed.

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